SNR Estimation Technique of the AWGN Channel by Second and Fourth-Order Moments (M_2M_4)

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Abstract

The goal is to find the "best" estimate of the SNR in a digital receiver with the least cost. Generally, SNR estimates are generated by averaging observable properties of the received signal over a number of symbols. The performances of various signal-to-noise ratio (SNR) estimation techniques reported in the literature are compared to identify the "best" estimator. The SNR estimators are investigated by the computer simulation of baseband binary phase-shift keying (PSK) signals in real additive white Gaussian noise (AWGN) and baseband M-ary PSK signals in complex Additive white Gaussian noise (AWGN). Some known estimator structures are modified to perform better on the channel of interest. It investigates, therefore, both types of estimators and quantifies by example the improvement in performance achievable by using known data rather than error-corrupted recovered data. Estimator structures for complex channels are examined.

Keywords: Signal-to-noise ratio (SNR), Phase-shift keying (PSK), Additive white Gaussian noise (AWGN), Phase Shift Keying (PSK).

1.0 Introduction

Modern wireless communication systems often require knowledge of the signal to noise ratio (SNR) at the receiver. For example, SNR estimates are typically employed in power control, mobile assisted handoff and adaptive modulation schemes. The rapid development of these applications in the last decade has led to an intense search for accurate and low complexity SNR estimators. The problem of SNR estimation may be considered for data-aided (DA) scenarios, where known transmitted data is used to facilitate the estimation process, and for non-data-aided (NDA) scenarios since the periodic transmission of known data limits system throughput. The basic problem was first introduced in the 1960s by [1] and [2]. However, decreasing hardware costs and increasing demands for pushing system performance to the achievable limits make an investigation of SNR estimation techniques timely.

The search for a good signal-to-noise ratio (SNR) estimation technique is motivated by the fact that various algorithms require knowledge of the SNR [3, 4] for optimal performance if the SNR is not constant. The performance of diverse systems may be improved if knowledge of the SNR is available. Past engineering practice has often used an estimation of the total signal-plus-noise power instead of estimation of the SNR since it is much easier to measure total power than the ratio of signal power to noise power (or noise power spectral density).

1.1 Quadrature phase shift keying (qpsk)

QPSK is the digital modulation technique. QPSK is a form of PSK in which two bits are modulated at once, selecting one of four possible carrier phase shifts; For example, the four possible pairs of bits can be represented 10, 00, 01 and 11 as follows:

$S_0\ (t) = \sqrt{2}A_c \cos\left(2\pi f_c t + \frac{\pi}{4}\right)$ For bit pair 11

$S_0\ (t) = \sqrt{2}A_c \cos\left(2\pi f_c t + \frac{3\pi}{4}\right)$ For bit pair 01

$S_0\ (t) = \sqrt{2}A_c \cos\left(2\pi f_c t - \frac{\pi}{4}\right)$ For bit pair 10

$S_0\ (t) = \sqrt{2}A_c \cos\left(2\pi f_c t - \frac{3\pi}{4}\right)$ For bit pair 00

Where, $0 \leq t \leq T$. That is, the carrier is transmitted with one of four possible phase values, $\pm\pi/4, \pm 3\pi/4$, with each phase corresponding to a unique pair of bits as shown in the figure.

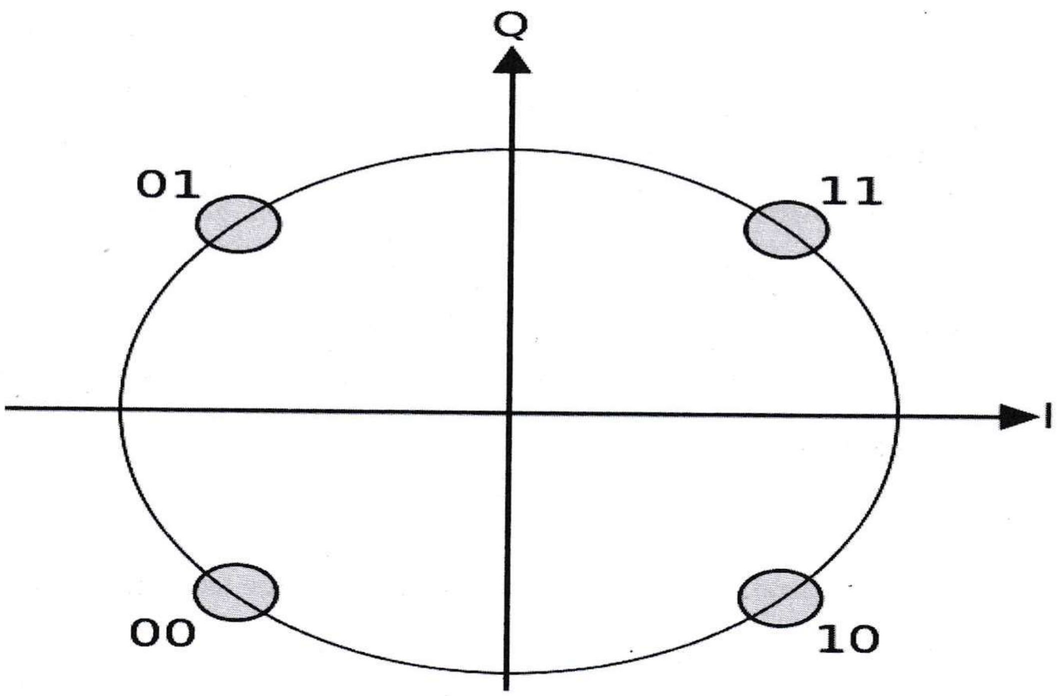

Fig.1.1. Constellation diagram of QPSK

QPSK perform by changing the phase of the In-phase (I) carrier from 0° to 180° and the Quadrature-phase (Q) carrier between 90° and 270°. This is used to indicate the four states of a 2-bit binary code. Each state of these carriers is referred to as a Symbol. QPSK is a widely used method of transferring digital data by changing or modulating the phase of a carrier signal. In QPSK digital data is represented by 4 points around a circle which correspond to 4 phases of the carrier signal. These points are called symbols. QPSK modulation consists of two BPSK modulations on in-phase and quadrature components of the signal.

2.0 Quadrature amplitude modulation (QAM)

QAM has fast become the dominant modulation mechanism for high-speed digital signals. QAM is a modulation technique which conveys data by changing some aspect of a carrier signal or the carrier wave (usually a sinusoid) in response to a data signal. In the case of QAM, the amplitude of two waves, 90° out of phase with each other (in quadrature) are changedd (modulated or keyed) to represent the data signal.

QAM is both an analog and a digital modulation scheme. It conveys two analog message signals or two digital bit streams, by changing (modulating) the amplitudes of two carrier waves, using the amplitude shift keying (ASK) digital modulation scheme or amplitude modulation (AM) analog modulation scheme. The two carrier waves, usually sinusoids are out of phase with each other by 90° and are thus called quadrature carriers or quadrature components hence the name of the QAM. The modulated waves are summed, and the resulting waveform is a combination of both PSK and ASK or (in the analog case) of phase modulation (PM) and amplitude modulation (AM). In the digital QAM case, a finite number of at least two phases and at least two amplitudes are used. PSK modulators are often designed using the QAM principle but are not considered as QAM since the amplitude of the modulated carrier signal is constant. QAM is used extensively as a modulation scheme for digital telecommunication systems. Arbitrarily high spectral efficiencies can be achieved with QAM by setting a suitable constellation size, limited only by the noise level and linearity of the communication channel.

2.1Constellation diagram of 16 -QAM

In 16- QAM modulation technique one sixteen possible signals is transmitted during each signaling interval T, with each signal uniquely related to pairs of bits. Each symbol contains four bits. The 16-QAM can be expressed as

$M = 2^n$

Where, $n = 4$ no. of bits of each symbol.

The 16-state quadrature amplitude modulation includes two I values and two Q values are used, yielding four bits per symbol. The constellation diagram of 16-QAM modulation technique is shown in the figure.

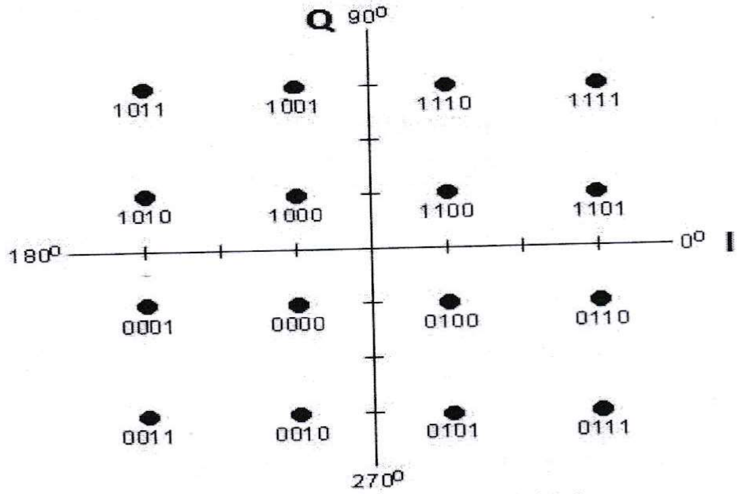

Fig. 2.1. Constellation diagram of 16-QAM .

Table 2.1. Carrier phase and amplitude of 16-QAM

Symbol Transmitted	Carrier Phase	Carrier Amplitude
0000	225°	0.33
0001	255°	0.75
0010	195°	0.75
0011	225°	1.0
0100	135°	0.33
0101	105°	0.75
0110	165°	0.75
0111	135°	1.0
1000	315°	0.33
1001	285°	0.75
1010	345°	0.75
1011	315°	1.0
1100	45°	0.33
1101	75°	0.75
1110	15°	0.75
1111	45°	1.0

The 16-QAM modulation offers twice higher spectral efficiency than QPSK and further reduces the required symbol rate to obtain the equivalent overall bit-rate, albeit, at the expense of an increased required OSNR, and worse performance in the linear and nonlinear transmission regime. A QPSK signal has 6.8dB lower required Optical Signal to Noise Ratio OSNR than 16-QAM signal for the same symbol rate of 28Gbaud, and also 3.8dB lower required OSNR for the same bit rate of 112Gbit/s. The 16-QAM will also have reduced tolerance towards nonlinearity than QPSK because of the presence of 3 intensity levels and, hence, higher peak-to-mean ratio. The DSP for 16-QAM signals is more complicated than for QPSK, in particular, adaptive equalization and carrier phase estimation. To date, few techniques have been proposed to generate 16-QAM signals. Perhaps the most prominent generation technique involves the synthesis of two 4-level electrical signals to drive the two arms of an I-Q modulator. In the simplest implementation, an IQ modulator is driven by two electrical signals with equally spaced amplitude levels over the linear part of its transfer function. In such configuration, the equally spaced electrical amplitude levels are linearly converted into the optical domain creating two 4-state amplitudes shift keyed (4-ASK) signals. An alternative to equally spaced amplitude levels is to pre-distort a 4-level electrical driving signal in order to drive an I-Q modulator over 2Vπ. Although in both cases the generated 4-ASK signal contains equally spaced optical levels, the latter configuration allows to exploit the full modulation depth of the modulator and suppress the noise in some of the 16-QAM symbols, albeit, at the expense of increased transmitter complexity. 16 QAM techniques are used in microwave digital radio, modems, DVB-C, DVB-T.

3.0 SNR estimation technique

The SNR of interest is the ratio of the discrete signal power to discrete noise power at the input to the decision device at the optimal sampling instants. In the following, "SNR" denotes this ratio of discrete powers. If a matched filter (MF) is employed at the receiver, the SNR ρ as defined here is related to the ratio of the symbol energy-to-noise power spectral densityE_s/N_o by $\rho = 2E_s/N_o$for real channels and$\rho = E_s/N_o$ for complex channels. Simulated QAM and QPSK signals in complex AWGN are used to investigate the performances of the second- and fourth-order moments (M_2M_4) estimator.

3.1 Second- and fourth-order moments (M_2M_4) estimator

An early mention of the application of second- and fourth-order moments to the separate estimation of carrier strength and noise strength in real AWGN channels was in 1967 by Benedict and Soong [6]. In 1993, Matzner [7] gave a detailed derivation of an SNR estimator which yielded similar expressions to those given in [6]. In 1994, Matzner et al. [8] rederived the estimator for real signals using a different approach. We sketch the derivation provided in [7] for complex channels below and then show how the estimator can be modified for application to real channels using the same approach. Let M_2 denote the second moment of y_n as

$$M_2 = E\{ y_n y_n^* \}$$
$$= SE\{|a_n|^2\} + \sqrt{SN}\, E\{a_n w_n^* \} + \sqrt{SN}\, E\{w_n a_n^* \} + NE\{|w_n|^2\} \quad (1)$$

And let M_4 denote the fourth moment of y_n as

$$M_4 = E\{ (y_n y_n^*)^2\}$$
$$= S^2E\{|a_n|^4\} + 2S\sqrt{SN}\,.(E\{|a_n|^2 a_n w_n^* \} + E\{|a_n|^2 w_n a_n^* \}) + SN.(E\{a_n w_n^*)^2\} + 4E\{|a_n|^2|w_n|^2\} + E\{(w_n a_n^*)^2\}) + 2N\sqrt{SN}(E\{|w_n|^2 a_n w_n^* \} + E\{|w_n|^2 a_n w_n^* \}) + N^2E\{|w_n|^4\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

Assuming the signal and noise are zero-mean, independent random processes, and the in-phase and quadrature components of the noise are independent, (1) and (2) reduce to

$$M_2 = S + N \quad \ldots\ldots\ldots\ldots\ldots\ldots(3)$$

and

$$M_4 = k_a S^2 + 4SN + k_w N^2 \quad (4)$$

respectively, where $k_a = E\{|a_n|^4\}/E\{|a_n|^2\}^2$ and $k_w = E\{|w_n|^4\}/E\{|w_n|^2\}^2$ are the kurtosis of the signal and the kurtosis of the noise, respectively. Solving for *S* and *N*, one obtains

$$S = \frac{M_2(k_w-2) \pm \sqrt{(4-k_a k_w)M_2^2 + M_4(k_a+k_w-4)}}{(k_a+k_w-4)} \quad (5)$$

and

$$N = M_2 - S \quad (6)$$

The estimator formed as the ratio of *S* to *N* is denoted the M_2M_4 estimator. As an example, for any *M*-ary PSK signal $k_a = 1$ and for complex noise $k_w = 2$ so that from equation (5) and (6) the result is

$$\rho M_2M_4, complex = \frac{\sqrt{2M_2^2 - M_4}}{M_2 - \sqrt{2M_2^2 - M_4}} \quad (7)$$

On the other hand, for the 16-QAM signal, $k_a = 1.32$ and for complex noise $k_w = 2$ so that from equation (5) and (6) the result is

$$\rho M_2 M_4, complex = \frac{\sqrt{2M_2^2 - M_4}}{M_2\sqrt{0.68} - \sqrt{2M_2^2 - M_4}} \quad (8)$$

The estimator is of the in-service type and has the advantage that carrier phase recovery is not required since it is based on the second and fourth moments of the signal. As a moments-based estimator, it does not use receiver decisions and so is not labeled as a DA estimator. In practice, the second and fourth moments are estimated by their respective time averages for both real and complex channels of *N* symbols as

$$M_2 = \frac{1}{N}\sum_{n=0}^{N-1} |y_n|^2 \quad (9)$$

And

$$M_4 = \frac{1}{N}\sum_{n=0}^{N-1} |y_n|^4 \quad (10)$$

3.2Proposed technique for 16-QAM

The constellation diagram and circle representation of 16-QAM technique are given below. The coordinates of first (inner) circle are $(1 \pm j)$ and$(-1 \pm j)$. The coordinates of second (middle) circle are $(1 \pm 3j)$, $(-1 \pm 3j)$, $(3 \pm j)$ and$(-3 \pm j)$ and the coordinates of the third (outer) circle are $(3 \pm 3j)$ and$(-3 \pm 3j)$.

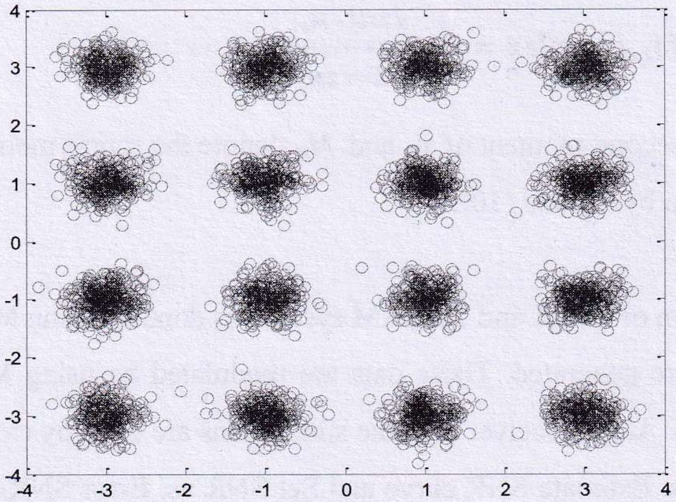

Fig. 3.1. Constellation diagram of 16-QAM.

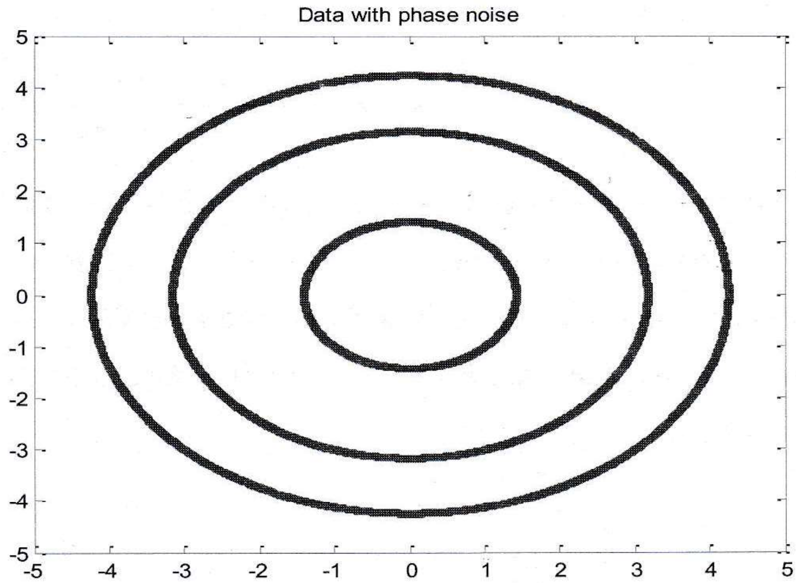

Fig.3.2. Circle representation of 16-QAM

When the SNR is high such as (17 or above) dB than the middle circle of 16-QAM is taken into consideration to estimate the SNR but the formula is used of QPSK

$$\rho M_2 M_4, complex = \frac{\sqrt{2M_2^2 - M_4}}{M_2 - \sqrt{2M_2^2 - M_4}} \qquad (11)$$

When the SNR is low such as (16 or below) dB than the formula of 16-QAM is taken into consideration to estimate the SNR

$$\rho M_2 M_4, complex = \frac{\sqrt{2M_2^2 - M_4}}{M_2\sqrt{0.68} - \sqrt{2M_2^2 - M_4}} \qquad (12)$$

Where M_2denote the second moment of y_n and M_4 denote the fourth moment ofy_n. the expression of these moments is given by (9) and (10).

4.0 Simulation setup

The simulation of QPSK and 16-QAM systems is done by using MATLAB software. At first, 50,000 random data are generated. These data are modulated by using MATLAB command. Then AWGN noise is added. At the receiver end, the simulations are done by existing formula & determine Error. The Set SNR vs. Estimate SNR curve and Set SNR vs. Error SNR curve are also constructed. The sequence of the function of simulation can be represented as follows:

RANDOM DATA GENERATION
MODULATION QPSK **or** 16-QAM
AWGN NOISE ADDITION
ESTIMATE SNR BY EXISTING FORMULA
ERROR CALCULATION
RESULTS

4.1 Simulation results

The simulation for SNR estimate of M_2M_4technique is used to reduce the Signal -to-Noise Ratio (SNR) error of transmitted signal. The simulation results for M_2M_4 SNR estimation technique are shown in the following figure.

4.1.1 Simulation result by using M_2M_4 QPSK estimator

The performance curves of Second- and Fourth-Order Moments (M_2M_4)QPSK technique shown in the following simulation result. From this simulation we can calculate the error in dB with respect to set SNR in dB vs. estimate SNR in dB curve and Error curve of this technique shown in the following fig. 4.1 and 4.2. This technique is preferable at low as well as high SNR. The estimate SNR is close to the set SNR hence error in this case close to zero db.

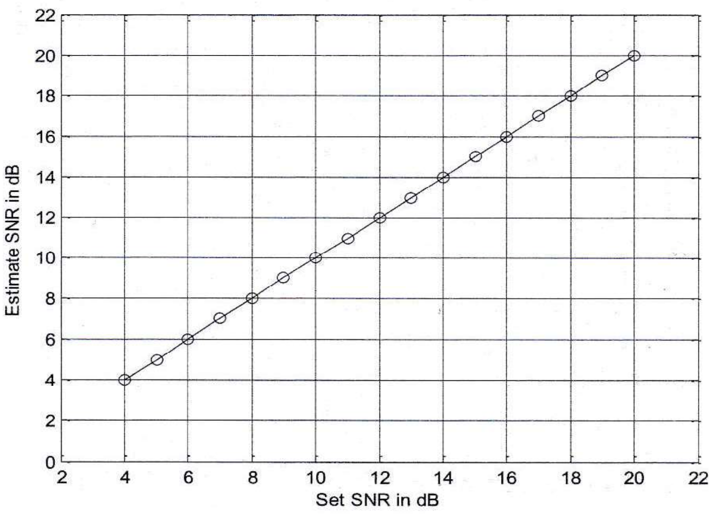

Fig. 4.1. Set SNR vs. Estimate SNR curve of M_2M_4QPSK technique

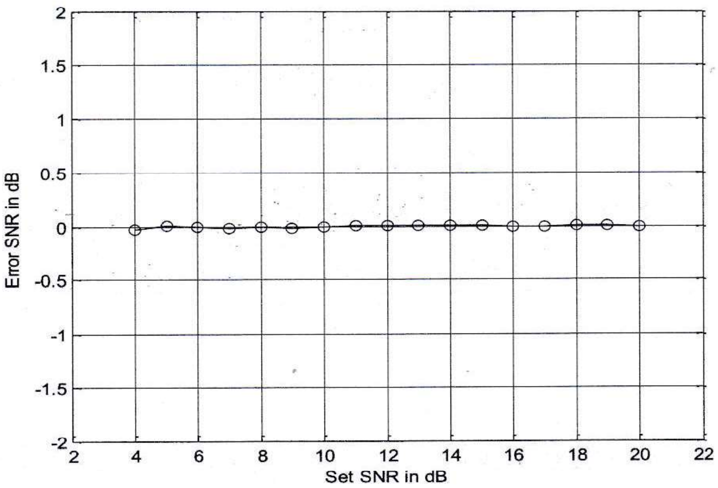

Fig. 4.2. Error curve of M_2M_4QPSK technique

4.1.2 Simulation result by using M_2M_4 16-QAM estimator

The performance curves of$M_2M_4$16-QAM technique shown in the following simulation result. From this simulation we can calculate the error in dB with respect to set SNR in dB vs. estimate SNR in dB curve and Error curve of this technique shown in the following fig. 4.3 and 4.4. This technique is preferable at the SNR which is less than 16 dB

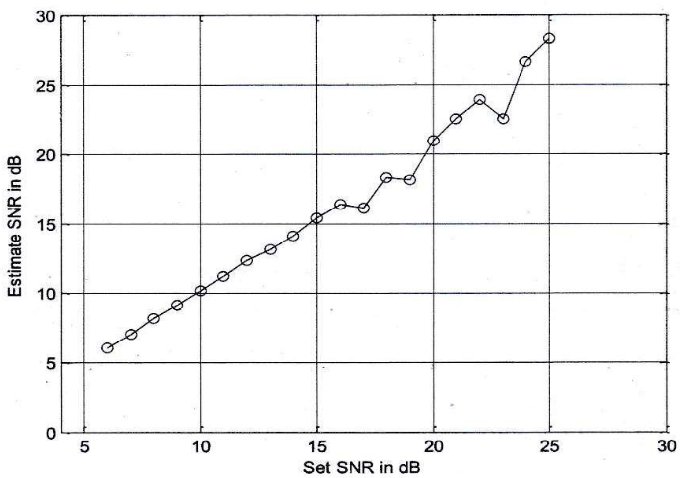

Fig. 4.3. Set SNR vs. Estimate SNR curve of $M_2M_4$16-QAM technique

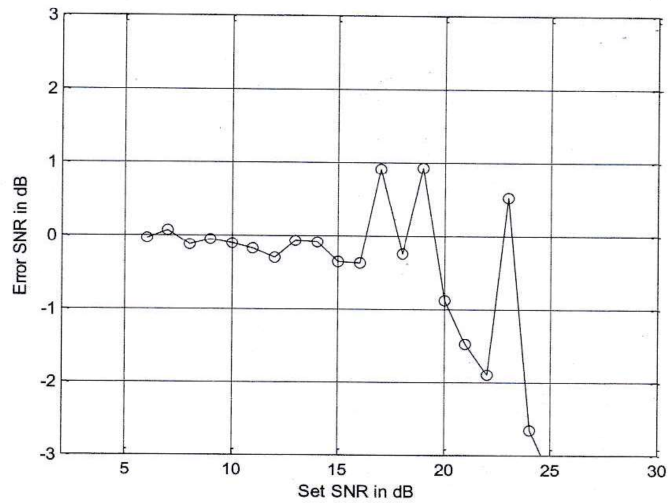

Fig. 4.4. Error curve of $M_2M_4$16-QAM technique

4.1.3 Simulation result by using $M_2M_4$16-QAM (inner circle) estimator

The performance curves of $M_2M_4$16-QAM (inner circle) technique shown in the following simulation result. From this simulation we can calculate the error in dB with respect to set SNR in dB vs. estimate SNR in dB curve and Error curve of this technique shown in the following fig. 4.5 and 4.6. This technique is not preferable at low as well as high SNR. The estimate SNR is much smaller than to the set SNR hence error in this case is very large.

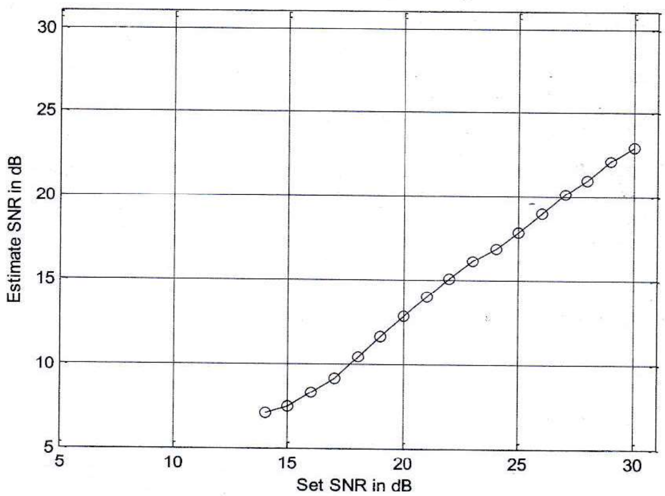

Fig. 4.5. Set SNR vs. Estimate SNR curve of $M_2M_4$16-QAM (inner circle) technique

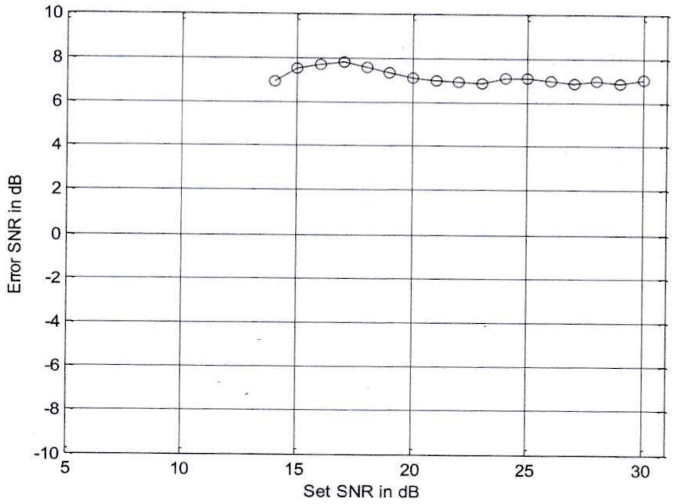

Fig. 4.6. Error curve of $M_2M_4$16-QAM (inner circle) technique

4.1.4 Simulation result by using $M_2M_4$16-QAM (outer circle) estimator

The performance curves of $M_2M_4$16-QAM (outer circle) technique shown in the following simulation result. From this simulation we can calculate the error in dB with respect to set SNR in dB vs. estimate SNR in dB curve and Error curve of this technique shown in the following fig. 4.7 and 4.8. This technique is not preferable at low as well as high SNR. The estimate SNR is much larger than to the set SNR hence error in this case is very large.

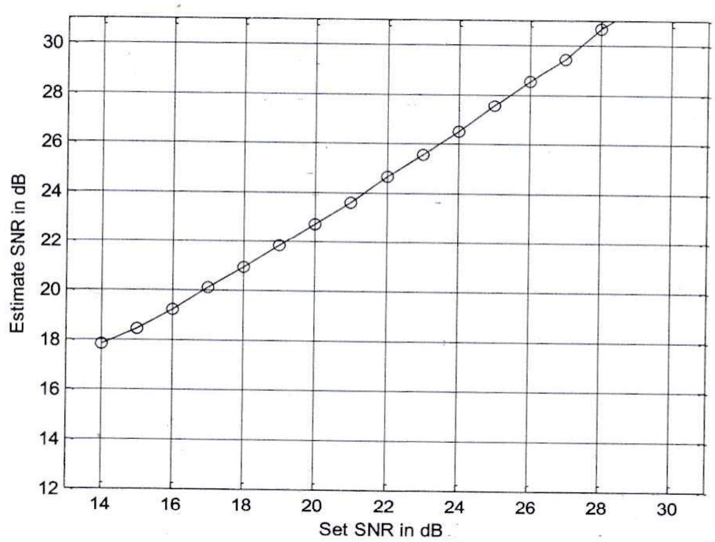

Fig. 4.7. Set SNR vs. Estimate SNR curve of $M_2M_4$16-QAM (outer circle) technique

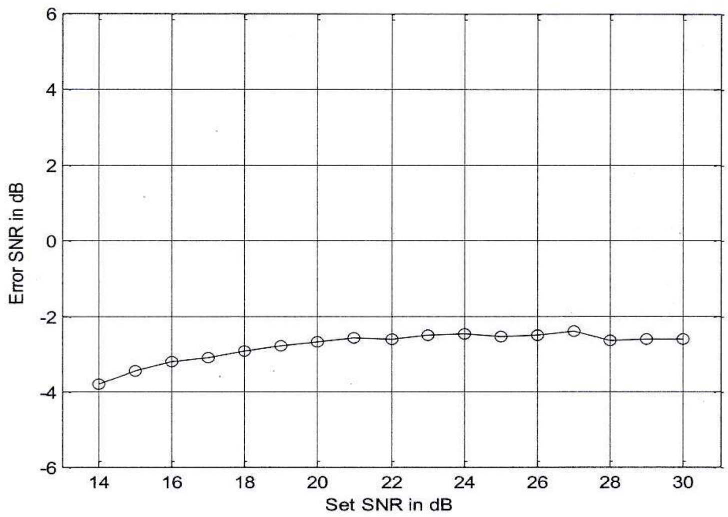

Fig. 4.8. Error curve of $M_2M_4$16-QAM (outer circle) technique

4.1.5 Simulation result by using $M_2M_4$16-QAM (middle circle) estimator:

The performance curves of $M_2M_4$16-QAM (middle circle) technique shown in the following simulation result. From this simulation we can calculate the error in dB with respect to set SNR in dB vs. estimate SNR in dB curve and Error curve of this technique shown in the following fig. 4.9 and 4.10. This technique is preferable at high SNR. The estimate SNR is close to the set SNR at high SNR (above 16dB) hence error in this case close to zero dB.

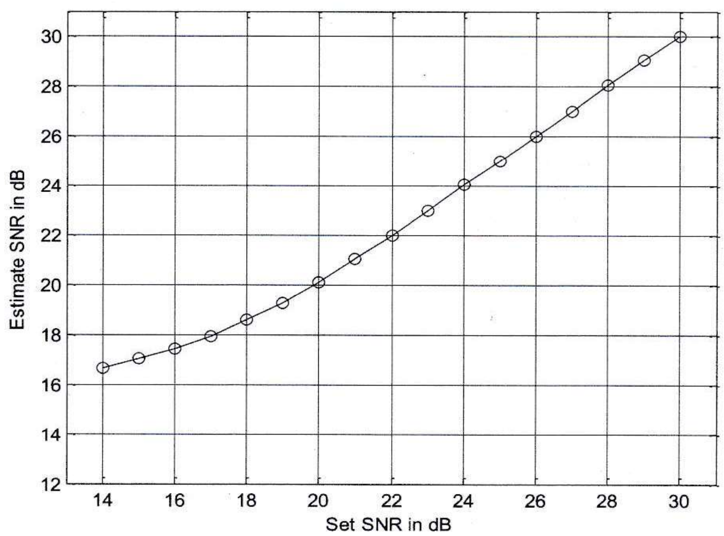

Fig. 4.9. Set SNR vs. Estimate SNR curve of $M_2M_4$16-QAM (middle circle) technique

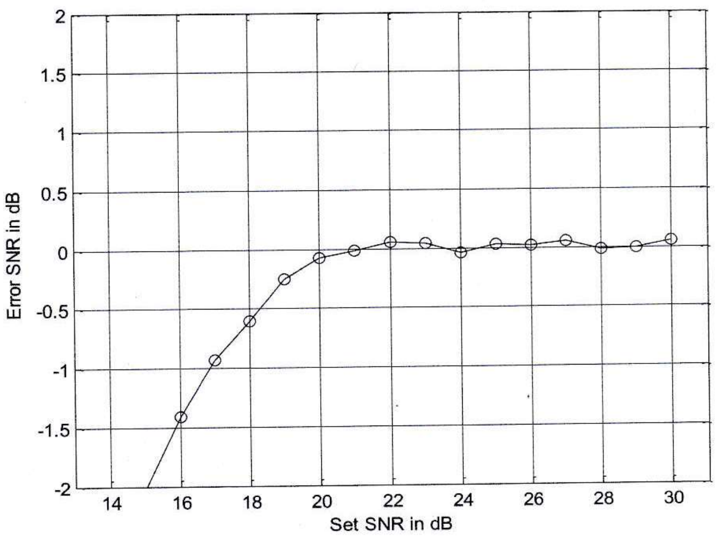

Fig. 4.10. Error curve of $M_2M_4$16-QAM (middle circle) technique

4.1.6 Simulation result of a proposed (hybrid) technique for 16-QAM:

The performance curves of proposed (hybrid) technique for 16-QAMshown in the following simulation result. From this simulation, we can calculate the error in dB with respect to set SNR in dB vs. estimate SNR in dB curve and Error curve of this technique shown in the following fig. 4.11 and 4.12. This technique is preferable at low as well as high SNR. The estimate SNR is close to the set SNR hence the error in this case close to zero dB.

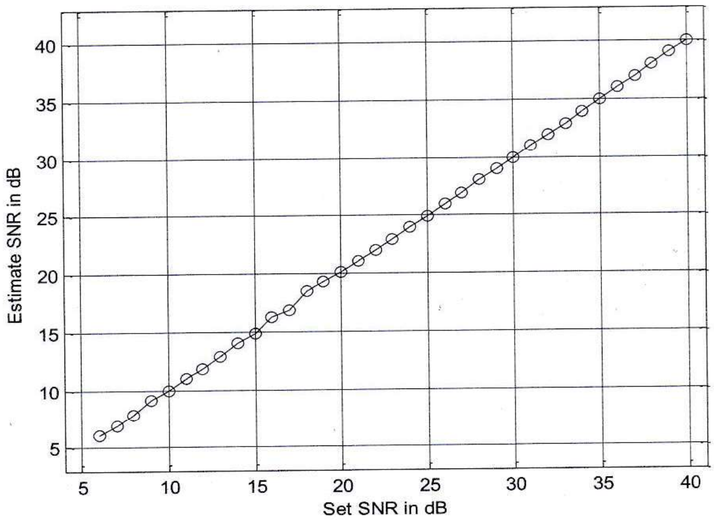

Fig. 4.11. Set SNR vs. Estimate SNR curve of hybrid technique

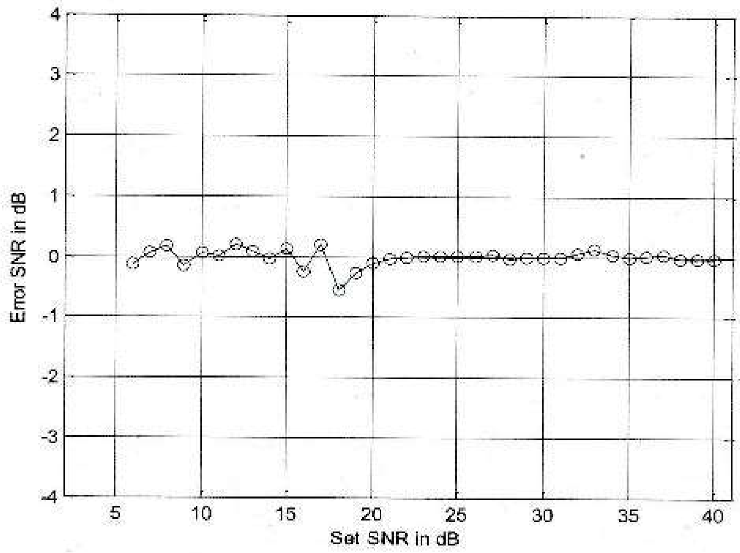

Fig. 4.12. Error curve of hybrid technique

5.0 Conclusion

This paper has considered the problem of SNR estimation in linear and time-invariant (LTI) AWGN channels. The estimator M_2M_4 is asymptotically efficient at high SNR. The M_2M_4 estimator is easy to implement, a hardware implementation ofthe M_2M_4 estimator is described in [13].The performance of the M_2M_4 estimators was analytically computed with and without channel polynomial order mismatch. Simulation results have been presented to investigate estimator performance for AWGN channels. Theoretical analysis has shown that the accuracy of SNR estimation should not degrade due to the channel's time variation. The M_2M_4 the estimator is based on higher-order moments, the comparison between M_2M_4 and proposed technique for 16-QAM is given below.

SNR estimation techniques	**SNR**	**Error**
M_2M_4 QPSK	Suitable at any SNR	Close to zero
M_2M_4 16-QAM	Suitable at low SNR (1dB-16dB)	Low at low SNR
M_2M_4(16-QAM middle circle)	Suitable at high SNR (above 17dB)	Low at high SNR
M_2M_4(16-QAM inner circle)	Not Suitable at any SNR	High
M_2M_4(16-QAM outer circle)	Not Suitable at any SNR	High
Proposed technique for 16-QAM	Suitable at high & low SNR	Close to zero

5.1 Future work

There remain several issues that can be used as starting points and/or central themes for future studies. For future work, it would be a good task to optimize the error and increase the modulation order of 16-QAM. We can also extension our analysis for higher order modulation such as 32-QAM, 64-QAM, and 128-QAM.

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